Chaotic sources and Percolation of strings

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Abstract

It is shown that different ways of interacting strings formed in high energy nucleus-nucleus collisions cause a different strength of the chaoticity parameter λ of Bose-Einstein correlations. In particular, in the case of percolation of strings, λ shows a peculiar dependence on the string density, very similar to the dependence of the fractional average cluster size. In both, the derivative on the string density is maximum at the critical point. The reasonable agreement with the existing experimental data indicates that percolation of strings can actually occurs.

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In the search of the possible Quark-Gluon-Plasma (QGP) state of matter there are several interesting studies [1, 2] on the space-time extension of the production region from which finally observed particles emerges, looking at the Bose-Einstein correlation (BEC) for two identical particles

$$C_2(Q^\mu) = 1 + \lambda f(Q^\mu) \tag{1}$$

$$Q^\mu = p_1^\mu - p_2^\mu \qquad f(0) = 1 \tag{2}$$

It is well known that the shape of the function $f(Q^\mu)$ can give information on the size of the source [2, 3]. In this paper we look at the correlation strength parameter λ, also called chaoticity, showing that also bears valuable information related to the dynamics of the multiparticle production and in particular to the possible formation of QGP. In particular, as λ counts the number of effective independent sources [4, 6], it will behave in a very different way depending on how interact these sources and therefore of the resulting number of effective independent sources.

We assume, as most of the models of soft hadronic interactions, that strings are formed among the partons of projectile and the target of the collision. The fragmentation of the strings is due to the formation of quark-antiquark or diquark-antidiquark pairs in different points. Each string can be considered as a totally chaotic source $\lambda = 1$ [7] (The possibility of some degree of coherence also has been considered in some version of fragmentation [8]). Each string has a transverse size πr^2, determined by the colour field streched between the colour charges of the partons placed at the extremes of the strings. As the energy and/or the size of projectile and target increases the number and density of strings increases, originating the interaction among strings which are not independent any longer. There are several possibilities of interaction. We will considerer three main cases:

a) String fusion [9] or colour rope [10] formation. Strings fuse as soon as their transverse positions come within a certain interaction area of the order of the string transverse dimension πr^2. The fusion of strings may take place only when their rapidity intervals overlap. The emerging string has the energy-momentum sum of the energy-momentum of the original strings and the same transverse size πr^2. The colour properties of the formed string is determined from the standard SU(3)-colour composition laws. The transverse size of the new string is the same as the one of the original strings.

b) Clusters of strings where the total transverse size is the geometrical one. Clusters with the same number of strings can have different sizes depending on the way of overlapping of the strings. In this case we are going to assume that the

colour field is SU(3) summed only in the overlapping regions [11]. Therefore one cluster should be considerer as several independent sources in most of the cases.

c) The clusters of strings have also the geometrical size but the colour-field is homogeneous all over the cluster area as water drops [12]. Each cluster can be considerer as a single source of incoherent (chaotic) production of particles.

In case a) of string fusion or colour rope there is not possibility of phase transition [13]. In cases b) and c) at high energy and/or large projectile and target when the string density reach a critical value η_c there is at least one path formed of overlapping strings through the transverse area of the collision. A second order phase transition takes place, the percolation of strings [14]. The critical value η_c of η,

$$\eta = \pi r^2 \frac{N}{\pi R^2} \tag{3}$$

takes the value 1.17-1.5 depending on the profile functions of the colliding nuclei used. N is the number of strings of transverse size πr^2 formed in a collision whose transverse area is πR^2.

Taking into account that each string is an incoherent source ($\lambda = 1$), it can be shown [15]

$$\lambda = \frac{n_S}{n_T} \tag{4}$$

where n_S are the number of pair particles produced in the same string in the rapidity and transverse momentum range fixed, and n_T is the total number of particles produced in the same kinematic range. To obtain equality (4) it is neglected the effects of B-E correlations on the single inclusive cross section. This fact, must be taken into account to correct the value of λ obtained from formula (4). We correct this value in the same way a some experiments do [16], obtaining a new value λ

$$\lambda = K_{spc}(\lambda_u + 1) - 1 \tag{5}$$

where λ_u is the uncorrected one and K_{spc} is the single particle correction which is evaluated for each collision and kinematics range in the same way described in reference [16]. To obtain (4) it is assumed that these are not BEC among pairs coming from different strings. This assumption is in agreement with L3, Delphi an ALEPH studies on BEC in W^+W^- experimental events produced in e^+e^- at LEP [17].

Qualitatively, it is clear that without any kind of interactions of strings $\lambda = 1$ and $\mu = n\mu_1$ where N, μ and μ_1 are the total number of strings, the average total multiplicity and the mean multiplicity of one string. In the case a) of string fusion,

the dominant term is

$$\lambda = \frac{1}{< M >} \qquad < M >=< \sum_n \nu_n > \tag{6}$$

where $< M >$ is the mean value of resulting strings, (clusters), and ν_n is the number of clusters formed from n original strings. As the probability of obtaining ν_n clusters is [13]

$$P(\nu_n) = \frac{c\,p^{N-M}}{\prod_{n=1}(\nu_n!(n!)^{\nu_n})} \prod_{k=1}^{M-1} (1-kp) \tag{7}$$

where c is a normalization constant, and p the fusion probability $p = r^2/R^2$ the mean values are

$$< \nu_n >= C_N^n p^{n-1}(1-p)^{N-n} \quad , \quad < M >= \frac{1}{p}(1-(1-p)^N) \tag{8}$$

In the *thermodynamical limit* $p \to 0$, $N \to \infty$, the relevant parameter is $\eta = Np$ and

$$\frac{< M >}{N} = \frac{1}{\eta}(1 - e^{-\eta}) = F(\eta)^2 \tag{9}$$

therefore

$$\lambda \to \frac{1}{NF(\eta)^2} \tag{10}$$

As $\mu = NF(\eta)\mu_1$, the quantity

$$\frac{\lambda\mu}{\mu_1} \to \frac{1}{F(\eta)} \tag{11}$$

only depends on η.

In the case b), denoting by n_i the number of regions where there are just overlapping of i strings, S_{ij} the area of the j-th region where there are just overlapping of i strings, $S_i = \sum_{j=1}^{n_i} S_{ij}$ the total area where there are overlapping of just i strings, $\sigma = \pi r^2$, and $\alpha = 2\arccos(R/2)$, we have in the same approximation as in (6)

$$\lambda = \frac{< \sum_{i=1}^{N} \sum_{j=1}^{n_i} i\frac{S_{ij}^2}{\sigma^2} >}{< (\sum_{i=1}^{N} \sqrt{i}\frac{S_i}{\sigma})^2 >} \tag{12}$$

using the results of reference [11], in the thermodynamical limit we have

$$\lambda = \frac{I}{NF(\eta)^2} \tag{13}$$

$$I = \int_0^2 dR\,R\frac{2}{\pi}(\alpha - \sin\alpha)\exp\left\{-2\eta\left[1 - \frac{1}{\pi}(\alpha - \sin\alpha)\right]\right\} \tag{14}$$

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And so

$$\frac{\lambda\mu}{\mu_1} = \frac{1}{F(\eta)}I \qquad (15)$$

depends only on η.

Finally in case c)

$$\lambda = \frac{< \sum_{i=1}^{N} \nu_i(\sqrt{i})^2 >}{< (\sum_{i=1}^{N} \nu_i\sqrt{i})^2 >} = \frac{N}{< (\sum_{i=1}^{N} \nu_i\sqrt{i})^2 >} \qquad (16)$$

which scales on η, and $\lambda \to 1$ as $\eta \to \infty$.

In fig. 1, it is plotted $\frac{\lambda\mu}{\mu_1}$ as a function of η for cases a) and b). In fig. 2 it is shown the behavior of λ on η for the case c). It is shown that in this case λ drops as η increases, for η less than 0.25. For further increases of η, λ increases even for η less than the critical percolation threshold η_c. This behaviour is similar to the average fractional cluster [18] size as a function of η. Also $\frac{d\lambda}{d\eta}$ vanishes at $\eta \approx \eta_c$. This behavior, contrary to the cases a) and b) is in qualitative agreement with the data as we will show below. To be more quantitative, we need to take into account energy-momentum conservations, and the energy-momentum and multiplicity of each cluster. We simulate the collisions using the framework of the string fusion model code [9]. In this code, it is know the transverse coordinates of the partons which form each string, and their energy-momentum distribution. As the transverse size of each string πr^2, we can located in the impact parameter plane, each string and therefore to know the clusters formed and their energy-momentum. We know that in the case c) a cluster formed from n strings on average will produce $\sqrt{n}$ times more particles than the produced in one string. This is the main information which we use in the simulation. The rapidity distribution of a decaying cluster formed with n strings is taken a gaussian with the proper normalization to take into account the mentioned factor $\sqrt{n}$. Instead gaussians we have used other reasonable parametrizations without any strong change in our results.

In figure 3 we present our results of λ versus η in the case c) evaluated for $y_{1cm} = y_{2cm} = 0.5$. The error bars represents the uncertainties due to different parametrizations of the string fragmentation.

It is seen that the shape is similar to the curve of figure 2. There are small violations of η scaling due to minor differences which can appear in the detailed kinematics of the strings formed in different nucleus-nucleus collisions, producing differences in the string density in the rapidity interval considered, and also due to finite size effects. In anycase these differences are less than 10%. Our evaluations were done at central rapidity. Notice that the λ value for a fixed collision depends

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on the rapidity range studied. In the extreme of the rapidity range less strings are formed and a higher λ is obtained.

The different experimental data are obtained in very different kinematics situations and also at very different centrality of the collisions what does uneasy the comparison and even the comparison among them. In anycase, data with ligth projectile as hadrons or Oxygen shows [5, 19] that λ falls as the multiplicity or the size of the target increases. For instance the values of λ for O-C, O-Cu, O-Ag and O-Au are [20] 0.92, 0.29, 0.22 and 0.16 respectively (These numbers corresponds to a centrality given by the average number of participants equal to 19.2, 39.5, 47.2 and 52.9 respectively. The rapidity range considered was $-1 \leq y_{lab} \leq 1$). The same experiment show some weaker decrease for different projectiles: λ=0.45, 0.32 and 0.33 for p, O and S respectively colliding against Au target. NA44 quoted a value of 0.56 for S-Pb central collisions (3% centrality) and 0.59 for Pb-Pb central collisions (15% centrality), (the range of pseudorapidity is $1.8 < \eta < 3.3$) [21]. In other kinematic situation NA44 quoted [16] $\lambda = 0.46$ for S-Pb ($3.1 < y < 4.3$). NA49 obtains [22] $\lambda = 0.42$ for central Pb-Pb collisions ($2.9 < y < 5.5$, $p_T < 0.6$).

The experimental situation on λ is not clear but some trends can be distinguised. Fist, a falling of λ with the multiplicity for low density of collisions. Second, as the density becomes higer λ stop of decreasing and increases. This trend is just the behaviour obtained for λ in the case of percolation strings as water drops. The forthcoming experiments at RHIC and LHC can confirm this behaviour by means of systematic studies of a range of projectiles and targets and for different centralities.

Acknowledgments

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Figure captions

Fig. 1. Dependence of the product of the chaoticity λ and the ratio between multiplicities as a function of η for string fusion (solid line) and case b) (dashed line).

Fig. 2. Dependence of λ on η for the case c) of percolation of strings.

Fig. 3. Dependence of λ on η for the case c) of percolation of strings in the MonteCarlo simulation. Points are K_{spc} corrected and correlations are calculated between identical pions. Nonfilled triangle corresponds to C-C minimum bias collisions at SPS energy; filled boxes to S-S minimum bias collisions at SPS, RHIC and LHC energies; nonfilled boxes to O-O central collisions ($b \leq 3.2$ fm.) at SPS, RHIC and LHC energies; filled triangles to S-S to central collisions ($b \leq 3.2$ fm.) at SPS, RHIC and LHC energies; stars to Ag-Ag central collisions ($b \leq 3.2$ fm.) at SPS, RHIC and LHC energies and diamonds to Pb-Pb central collisions ($b \leq 3.2$ fm.) at SPS, RHIC and LHC energies.
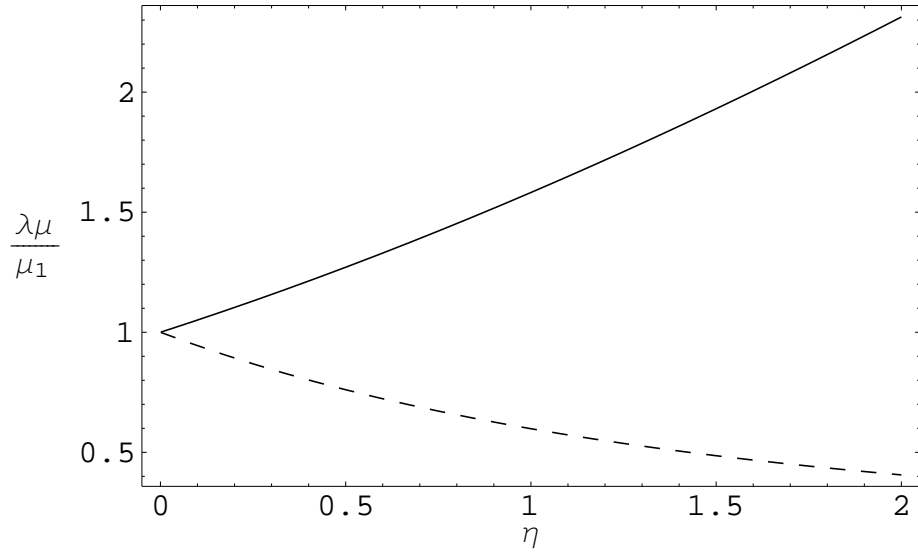
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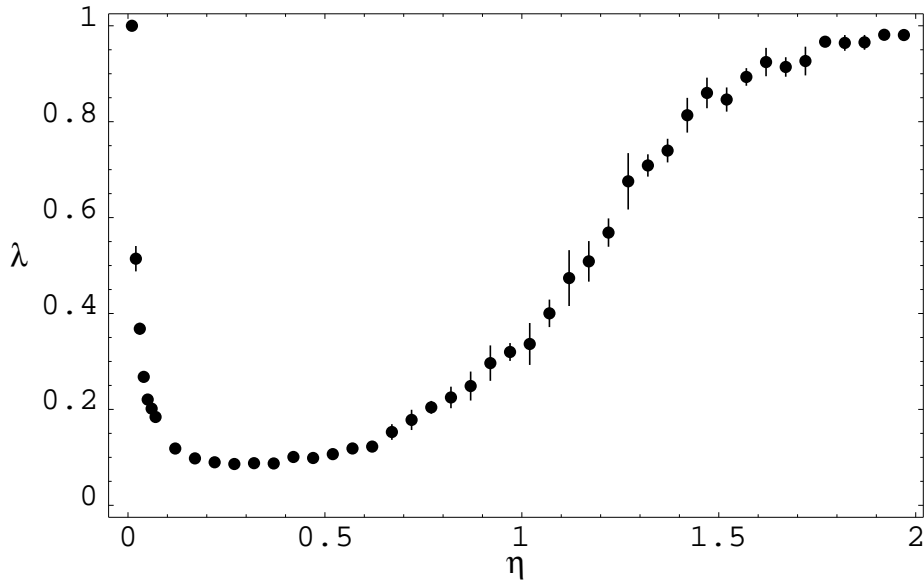

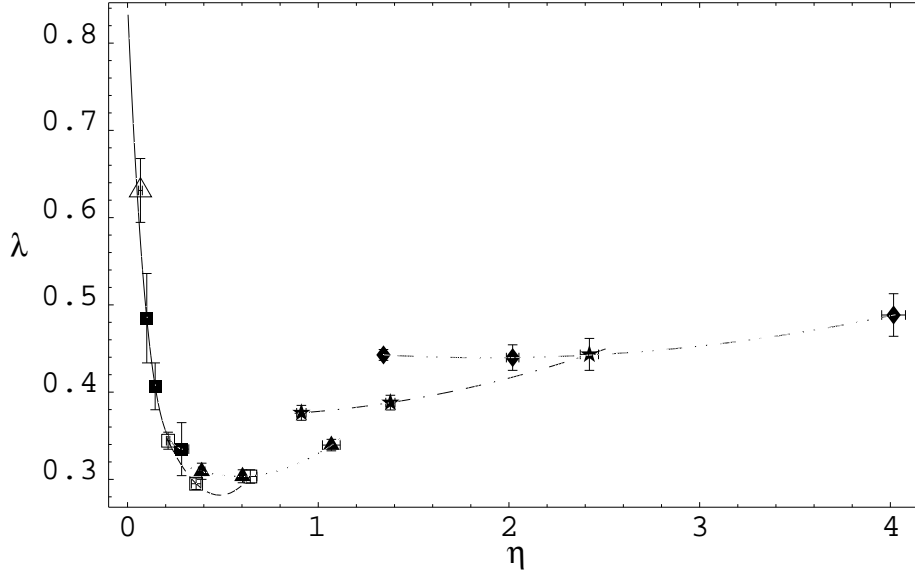Figure 2:   Dependence of λ on η for the case c) of percolation of strings.

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